WIDEBAND, TOTALLY COUPLED DIRECTIONAL TRANSFORMERS

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Abstract

Single-section and two-section nonsymmetrical directional couplers are introduced, providing large impedance transformation ratios, total matching, perfect directivity, almost unity coupling and flat delay across a 100% relative bandwidth. Design parameters are given and alternative physical implementations are discussed.

Total Matching and Perfect Directivity

The most interesting and valuable conclusion of the recent, extensive theoretical work [1], [2], [3], [4], [5] describing wave propagation and mutual coupling upon nonsymmetrical parallel coupled lines in non-homogeneous media is the discovery of a new general condition for simultaneous total matching and perfect directivity.

This condition was obtained by careful inspection and analysis of the closed-form expressions of the six different entries of the 4 × 4 scattering matrix of the nonsymmetrical and asynchronous coupled lines, as derived [5] for the case of different resistive terminations Z_o^a and Z_o^b connected respectively at either end of the two lines.

These expressions contain the parameter $\alpha = \frac{Z_0^a}{Z_0^b}$, besides the intrinsic line parameters θ_e, θ_o, R_3 and the normalized even-mode admittance y_{oe}^a and the normalized odd-mode impedance z_{oo}^a of line a. In these expressions the already introduced congruence condition is assumed to be satisfied [2], [3].

Perfect directivity and total matching is obtained for $\alpha = \frac{1}{R_3}$ and $y_{oe}^a = z_{oo}^a = 1.0$, regardless of the mode velocity ratio $R_v = \frac{\theta_o}{\theta_e}$. The condition $y_{oe}^a = z_{oo}^a = 1.0$ implies $y_{oe}^b = z_{oo}^b = 1.0$ and means physically that both lines are matched to their relative terminations for both modes.

As a consequence no discontinuity exists for either mode at the four ports, a condition which would imply zero coupling for lines in a homogeneous medium.

In a non-homogeneous medium, however, relative phase rotation of one mode with respect to the other occurs, because of the differing mode velocities v_e and v_o. This relative phase rotation introduces power transfer, from one line to the other, continuously distributed along the direction of wave propagation.

Single - Section Directional Transformers

By adding the condition $R_v = \frac{\theta_o}{\theta_e} = 2.0$ to the already given conditions of total matching and perfect directivity, a four-port is obtained having the scattering matrix:

$$\left|S\right| = \begin{vmatrix} 0 & S_{12} & 0 & S_{14} \\ S_{12} & 0 & S_{14} & 0 \\ 0 & S_{14} & 0 & S_{34} \\ S_{14} & 0 & S_{34} & 0 \end{vmatrix} \qquad (1)$$

where:

$$S_{12} = \frac{1}{1+R_3}\left[(1 + R_3 \cos\theta_e - 2\sin^2\theta_e) - - j\sin\theta_e\,(R_3 + 2\cos\theta_e)\right] \qquad (2)$$

$$S_{34} = \frac{1}{1+R_3}\left[(R_3 + \cos\theta_e - 2 R_3 \sin^2\theta_e) - - j\sin\theta_e\,(1 + 2R_3\cos\theta_e)\right] \qquad (3)$$

$$S_{14} = -\frac{\sqrt{R_3}}{1+R_3}\left[(1 - \cos\theta_e - 2\sin^2\theta_e) + + j\sin\theta_e\,(1 - 2\cos\theta_e)\right] \qquad (4)$$

It is interesting to notice that:

$$\left|S_{12}\right| = \left|S_{34}\right| = \frac{1}{1+R_3}\sqrt{1 + R_3^2 + 2R_3\cos\theta_e} \qquad (5)$$

But:

$$\angle S_{12} = \operatorname{atan}\left[-\frac{\sin\theta_e\,(R_3 + 2\cos\theta_e)}{1 + R_3\cos\theta_e - 2\sin^2\theta_e}\right] \neq$$

$$\neq \angle S_{34} = \operatorname{atan}\left[-\frac{\sin\theta_e\,(1 + 2R_3\cos\theta_e)}{R_3 + \cos\theta_e - 2R_3\sin^2\theta_e}\right] \qquad (6)$$

Also:

$$\left|S_{14}\right| = \pm \frac{2\sqrt{R_3}}{1 + R_3} \sin \frac{\theta_e}{2}$$

$$(\text{Max at} \; : \; \theta_e = \pi, \; \theta_o = 2\pi) \tag{7}$$

$$\angle S_{14} = \text{atan} \frac{\sin \theta_e \; (1 - 2 \cos \theta_e)}{1 - \cos \theta_e - 2 \sin^2 \theta_e} \tag{8}$$

The corresponding delays have peculiar properties:

$$\text{DEL } (S_{12}) = \frac{\partial}{\partial\omega} \angle S_{12} =$$

$$= - \frac{\ell}{v_e} \left[1 + \frac{1 + R_3 \cos \theta_e}{1 + R_3^2 + 2 R_3 \cos \theta_e}\right] \tag{9}$$

$$\text{DEL } (S_{34}) = \frac{\partial}{\partial\omega} \angle S_{34} =$$

$$= - \frac{\ell}{v_e} \left[1 + \frac{R_3^2 + R_3 \cos \theta_e}{1 + R_3^2 + 2 R_3 \cos \theta_e}\right] \tag{10}$$

$$\text{DEL } (S_{14}) = \frac{\partial}{\partial\omega} \angle S_{14} = - \frac{3}{2} \frac{\ell}{v_e} = \underline{\text{Constant}} \tag{11}$$

$$\text{DEL } (S_{12}) + \text{DEL } (S_{34}) = - 3 \frac{\ell}{v_e} =$$

$$= 2 \cdot \text{DEL } (S_{14}) = \underline{\text{Constant}} \tag{12}$$

These properties are illustrated by Figs. 1, 2, 3, which represent the behavior of a single-section directional transformer with $R_3 = 4.0$. This specific situation implies $k_L = - k_c = - \frac{1}{3}\sqrt{\frac{2}{3}} = -0.272165527$, a condition exactly realizable only in a 4-conductor-plus-ground system with lines a and b balanced with respect to ground.

Two - Section Directional Transformers

Largely increased midband coupling and wideband response may be obtained by cascading two identical single-section directional transformers as the one described by Figs. 1, 2, 3 with interposed phase-equalizers E_1 and E_2 (Fig. 4) having a delay approximating DEL (S_{34}), and DEL (S_{12}) respectively. The coupling properties of this configuration are described by Fig. 5 where the solid lines refer to a 1:4.0 impedance transformation ratio. As may be seen, total coupling is obtained at two different points of the passband, which extends, with a $\pm$ 0.17 dB ripple to cover a relative bandwidth of $\simeq$ 100%.

Maximally-flat response is obtained for an impedance transformation ratio of

$$R = \frac{\sqrt{2} + 1}{\sqrt{2} - 1} = 5.828427125$$

as shown by the dashed lines in Fig. 5.

The obtained bandwidths are comparable to that of an 8-section maximally flat stepped-impedance transformer or that of a 3 dB, $\pm$ 0.5 dB ripple three-section directional coupler.

Cross - Section Geometry

The implied condition $k_L = - k_c$, necessary for total match and perfect directivity, may be obtained with balanced lines with "diagonal excitation" and suitably located dielectric slabs [6] (Fig. 6).

Alternatively, contrawound helices or broadside-coupled contrawound meander lines may be considered.

A study is being carried out to develop methods for the geometrical design of the cross-section from the given impedances and transformation ratio.

References

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[2] R. A. Speciale, Even- and Odd-Mode Waves for Non-symmetrical Coupled Lines in Non-Homogeneous Media," IEEE Trans., MTT (submitted for publication).

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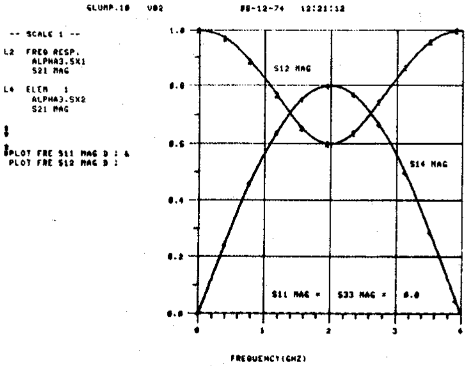

Figure 1.

Amplitude responses of a 1:4.0 single-section directional transformer. Curve 2 represents the transmission down line a from port 1 to port 2. Curve 4 corresponds to the signal coupled from line a into b, at a 4 times higher impedance level and represents the response from port 1 to port 4.

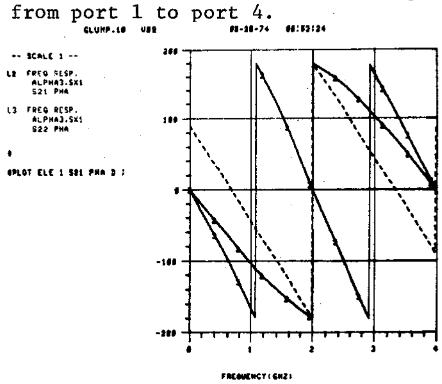

Figure 2.

Phase responses of a 1:4.0 single section directional transformer. Curve 2 is the phase rotation down line a, from port 1 to port 2. Curve 3 is the phase rotation down line b, from port 3 to port 4 (different from the phase rotation from port 1 to port 2). The dashed line is the phase rotation of the codirectionally coupled signal from port 1 to port 4, transformed in impedance by a factor 4.0. This last phase rotation is absolutely linear with frequency.

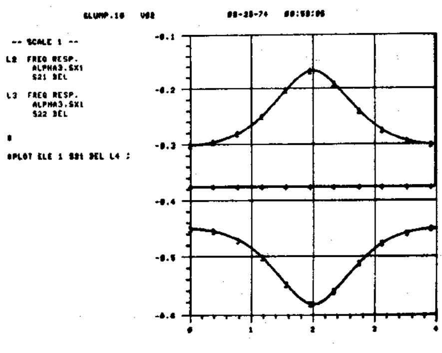

Figure 3.

Delay responses of a 1:4.0 single section directional transformer. Curve 2 is the delay down line a, from port 1 to port 2. Curve 3 is the delay down line b, from port 3 to port 4. The horizontal line 4 represents the absolutely constant delay of the codirectionally coupled signal from port 1 to port 4. The most peculiar property of the three delays is that the delay from port 1 to port 2 and the delay from port 3 to port 4 add up to twice the constant delay from port 1 to port 4.

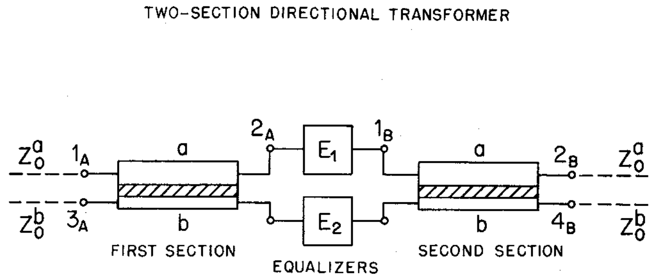

Figure 4.

Network configuration of the two-section directional transformer. Two identical single-section directional transformers are mutually cascaded with the low-impedance line a and the high-impedance line b on the same side. Phase equalizers E_1 and E_2 are interposed between ports 2_A and 1_B, 4_A and 3_B, approximating the delays DEL (S_{34}) of S_{34} and DEL (S_{12}) of S_{12} respectively. Physical implementations of these equalizers are known although not described in this paper.

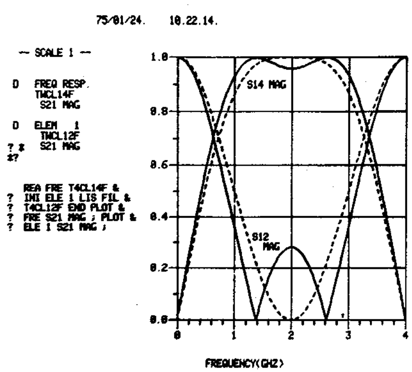

Figure 5.

Amplitude responses of the 1:4.0 (solid lines) and 1 to $(\sqrt{2}+1)/(\sqrt{2}-1)$ (dashed lines) two-section directional transformers. The lines labeled S_{14} represent the responses from port 1_A to port 4_B, those labeled S_{12} represent the responses from port 1_A to port 2_B. These responses are complementary in terms of power being: $|S_{12}|^2 + |S_{14}|^2 = 1.0$.

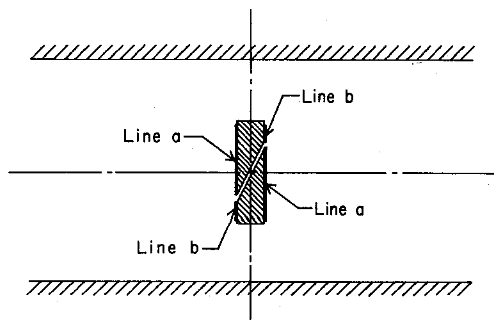

Figure 6.

Four-conductor plus ground system with dielectric slabs required for the exact realization of the $k_L = - k_c$ condition. The geometry shown is being designed for a 2 × 25 ohm line a and a 2 × 100 ohm line b.